

**Problem 2 : Solution – Jumping Beads - a model for phase transitions and instabilities (10 points)**

**Part A. Critical driving amplitude (3.3 points)**

**A1 (1.2 pts)**

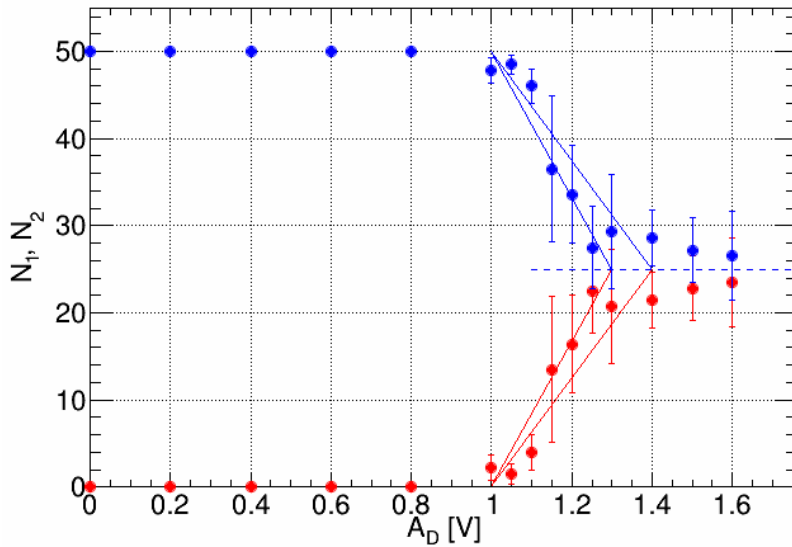
Total number of seeds:  $N_0 = 50$ .

Number of readings:  $n = 6$ .

$A_D$ , [V]	$N_1$						$\bar{N}_1 = \frac{1}{n} \sum_{i=1}^n N_1^i$	$\bar{N}_2 = N_0 - \bar{N}_1$	$\sigma = \sqrt{\frac{\sum_{i=1}^n (N_i - \bar{N})^2}{n-1}}$	$SE = \frac{\sigma}{\sqrt{n}}$
1.00	1	5	2	1	2	2	2.2	47.8	1.5	0.6
1.05	1	0	2	3	1	2	1.5	48.5	1.1	0.5
1.10	4	4	1	7	3	5	4.0	46.0	2.0	0.8
1.15	26	5	18	7	18	7	13.5	36.5	8.4	3.4
1.20	13	16	27	12	17	13	16.4	33.7	5.6	2.3
1.25	26	28	22	22	14	23	22.5	27.5	4.8	2.0
1.30	27	24	8	22	22	21	20.7	29.3	6.6	2.7
1.40	22	18	17	23	23	25	21.4	28.7	3.2	1.3
1.50	19	27	27	24	19	21	22.8	27.2	3.7	1.5
1.60	27	15	23	23	23	30	23.5	26.5	5.1	2.1

Plot the data in the graph A2.

**A2 (1.1 pts)**



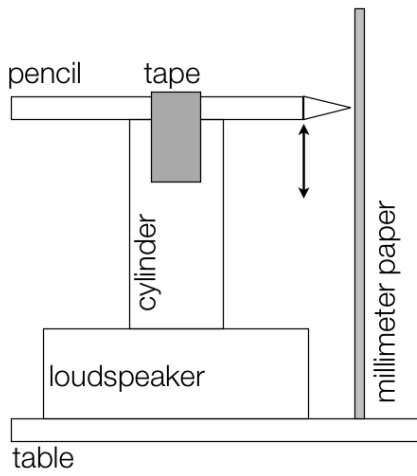
Error bars represent either standard deviation ( $\sigma$ ) or standard error (SE).

**A3 (1.0 pts)**

$A_{D,crit} = (1.25 \pm 0.05)$  V

Part B. Calibration (3.2 points)

**B1 (0.5 pts)**

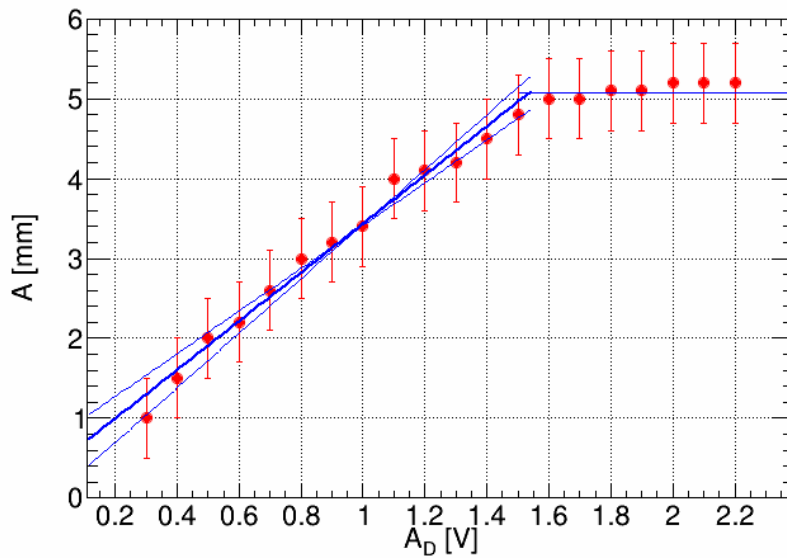


**B2 (0.8 pts)**

$A_D$ [V]	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1
$A$ [mm]	1.0	1.5	2.0	2.2	2.6	3.0	3.2	3.4	4.0	4.1	4.2	4.5	4.8	5.0	5.0	5.1	5.1	5.2	5.2

Instrumental error  $\pm 0.5$  mm.

**B3 (1.0 pts)**



**B4 (0.8 pts)**

$$A = k_0 + k_1 \times A_D,$$

where:

$$k_0 = 0.2 \text{ [mm]}, \quad k_1 = 3.1 \text{ [mm/V]}$$

**B5 (0.1 pts)**

$$A_{\text{crit}} = (4.4 \pm 0.1) \text{ mm}$$

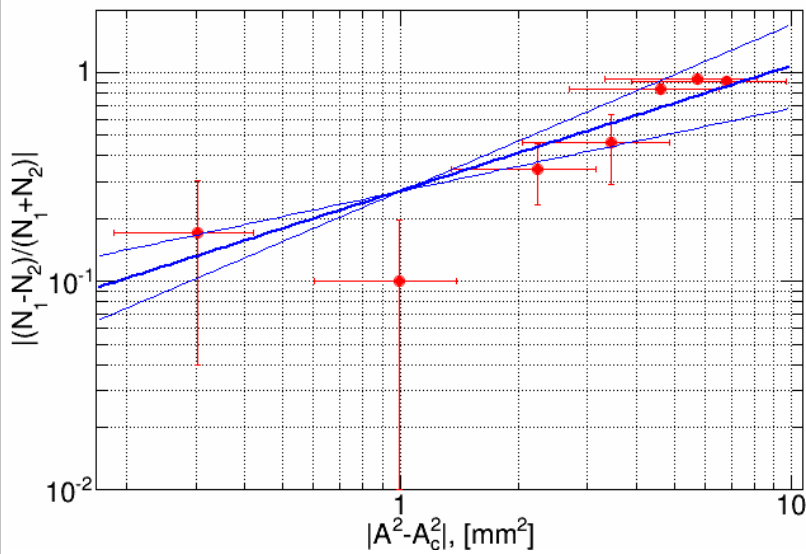
Part C. Critical exponent (3.5 points)

**C1 (1.1 pts)**

$A_D$ , [V]	$A$ , [mm]	$ \frac{N_1-N_2}{N_1+N_2} $	$ A^2 - A_0^2 $
1.00	3.5	0.91	6.8
1.05	3.6	0.94	5.7
1.10	3.8	0.84	4.6
1.15	3.9	0.46	3.5
1.20	4.1	0.35	2.2
1.25	4.2	0.10	1.0
1.30	4.4	0.17	0.3
1.40	4.7	0.15	
1.50	5.0	0.09	
1.60	5.3	0.06	

Plot the data in the graph C2.

**C2 (1.0 pts)**



**C3 (1.4 pts)**

$y = a \cdot x^b$ , where  $x = |A^2 - A_0^2|$ ,  $y = |\frac{N_1-N_2}{N_1+N_2}|$ .

Critical exponent  $b = 0.6 \pm 0.2$ .